

π IN THE SKY¹¹

ANSWER KEY



RECEIVER RIDDLE

How many kilometers ahead along Earth's orbit did the team need to aim the laser?

- Rearrange the distance formula to solve for time (t) and compute the length of time it took the transmission, traveling at the speed of light, to reach Earth.

$$D = rt \Rightarrow t = D/r$$

$$t = (30,199,000 \text{ km}) / (299,792 \text{ km/s}) \approx 101 \text{ seconds}$$

- Use the formula for circumference of a circle to compute the circumference of Earth's orbit.

$$C = 2\pi r = 2 \cdot \pi \cdot 149,000,000 \text{ km} \approx 936,194,611 \text{ km}$$

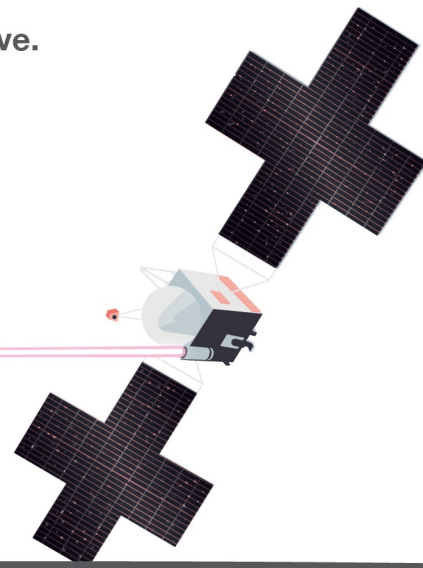
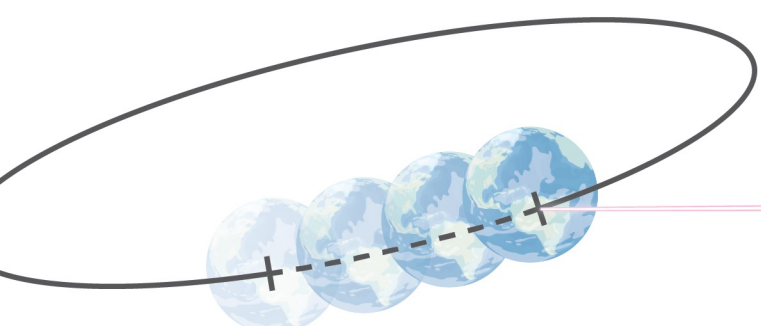
- Rearrange the distance formula to solve for rate (r) and convert units to compute Earth's rate of travel in kilometers per second.

$$D = rt \Rightarrow r = D/t$$

$$((936,194,611 \text{ km}) / (1 \text{ year})) (365.24 \text{ days} / 1 \text{ year}) (24 \text{ hours} / 1 \text{ day}) (60 \text{ min} / 1 \text{ hour}) (60 \text{ sec} / 1 \text{ min}) \approx 29.67 \text{ km/s}$$

- Use the distance formula once again to compute the distance Earth will have traveled during the time it took the transmission to arrive.

$$D = rt \approx (29.67 \text{ km/s}) \cdot (101 \text{ s}) \approx \mathbf{3,000 \text{ km}}$$



DARING DEFLECTION

Use Kepler's third law to calculate the semi-major axis (a) of the new orbit.

- Rearrange Kepler's third law equation to solve for the semi-major axis.

$$a = \sqrt[3]{(T / 2\pi)^2 \cdot GM}$$

$$a = \sqrt[3]{(40,980 \text{ sec} / 2\pi)^2 \cdot ((6.674 \cdot 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \cdot (5.643 \cdot 10^{11} \text{ kg}))}$$

$$a \approx \mathbf{1,170 \text{ meters}}$$

Calculate Dimorphos' apoapsis and periapsis.

- Plug in the given value for e and the calculated value for a.

$$\text{apoapsis} \approx 1,170 \text{ m} (1 + 0.02) \approx \mathbf{1,193 \text{ meters}}$$

$$\text{periapsis} \approx 1,170 \text{ m} (1 - 0.02) \approx \mathbf{1,147 \text{ meters}}$$

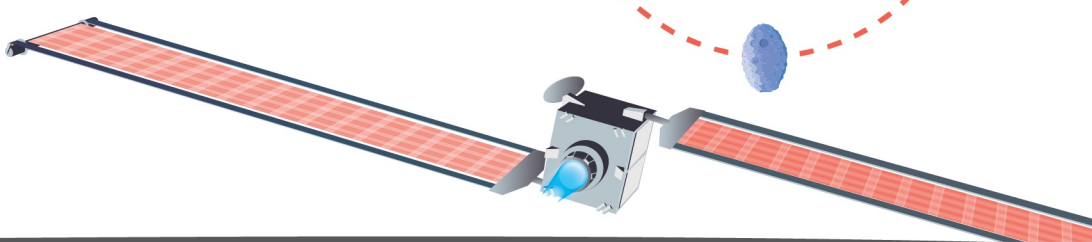
Compare the new elliptical orbit to the circular orbit.

- Convert the distances in meters to kilometers and compare the orbits' measurements.

$$\text{apoapsis: } 1,193 \text{ m} \approx 1.19 \text{ km}$$

$$\text{periapsis: } 1,147 \text{ m} \approx 1.15 \text{ km}$$

All points on the original circular orbit are equidistant from the central mass. Dimorphos' new elliptical orbit puts it at different distances from Didymos throughout its orbit, as shown by the apoapsis and periapsis calculations.



ORBIT OBSERVATION

How many orbits does NISAR execute in one day?

- Determine Earth's circumference using the given radius and the formula for circumference of a sphere.

$$2\pi r \approx 2(3.14)(6,371 \text{ km}) \approx 40,030 \text{ km}$$

- Use twice the width of the ground track to calculate the number of swaths needed to cover the entire globe, noting that the ground track passes the equator twice per orbit.

$$40,030 \text{ km} / 462 \text{ km} \approx 86.65 \text{ swaths}$$

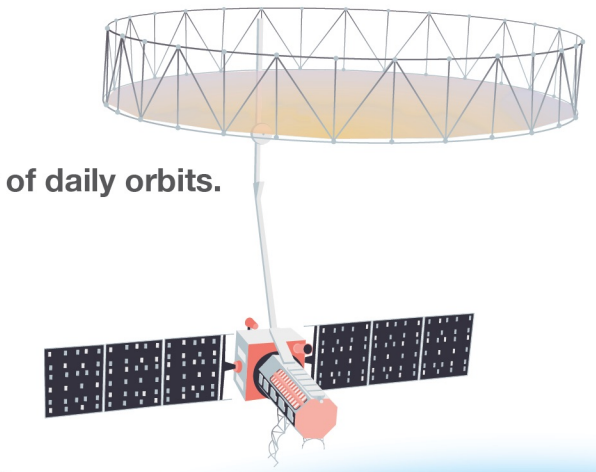
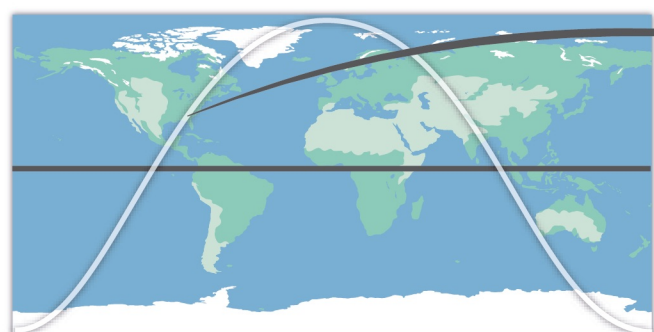
- Divide the swaths by the number of days it takes to map Earth once to get the number of orbits per day.

$$86.65 \text{ swaths} / 6 \text{ days} \approx \mathbf{14.4 \text{ orbits/day}}$$

How much data is produced per orbit on average?

- Divide the total data collected per day by the number of daily orbits.

$$85 \text{ TB/day} / 14.4 \text{ orbits/day} \approx \mathbf{5.9 \text{ TB per orbit}}$$



231 km ground track

MOON MAPPERS

How far does each rover have to drive to survey its portion of the Moon's surface?

- Convert radians to degrees, then use the Pythagorean theorem to find the swath width.

$$(\pi/2) \cdot (180^\circ/\pi) = 90^\circ$$

$$a^2 + b^2 = c^2 \Rightarrow (2 \text{ m})^2 + (2 \text{ m})^2 = c^2 \Rightarrow c = 2\sqrt{2} \text{ m}$$

- Determine the number of swaths the rovers need to drive to map the entire square.

$$20 \text{ m} / 2\sqrt{2} \text{ m} = 5\sqrt{2} \approx 7.07 \text{ swaths, round up to 8 swaths to ensure complete coverage}$$

- Use the Pythagorean theorem to determine the sensor triangle's altitude length.

$$a^2 + b^2 = c^2 \Rightarrow a^2 + (\sqrt{2} \text{ m})^2 = (2 \text{ m})^2 \Rightarrow a = \sqrt{2} \text{ m}$$

- Determine the horizontal drive distance for the first swath.

$$\sqrt{2} \text{ m} + 20 \text{ m} - \sqrt{2} \text{ m} = 20 \text{ m}$$

- Determine the vertical distance the rover must drive to position itself for the second swath. Note that the rover rotating in place does not add drive distance.

$$\sqrt{2} \text{ m} + \sqrt{2} \text{ m} = 2\sqrt{2} \text{ m}$$

- Determine the horizontal drive distance for the second swath, then add it to the vertical distance. The result is the drive distance for each subsequent swath.

$$20 \text{ m} - 2\sqrt{2} \text{ m}$$

$$20 \text{ m} - 2\sqrt{2} \text{ m} + 2\sqrt{2} \text{ m} = 20 \text{ m}$$

- Compute the total distance to be driven by all three rovers, then divide by three.

$$(20 \text{ m})(8) = 160 \text{ m}$$

$$(160 \text{ m}) / 3 \approx 53.3 \text{ m, round up to } \mathbf{54 \text{ m}}$$

