π IN THE SKY⁹

Answer Key

Lunar Logic

How much area does one of Lunar Flashlight's lasers cover in a single pulse?
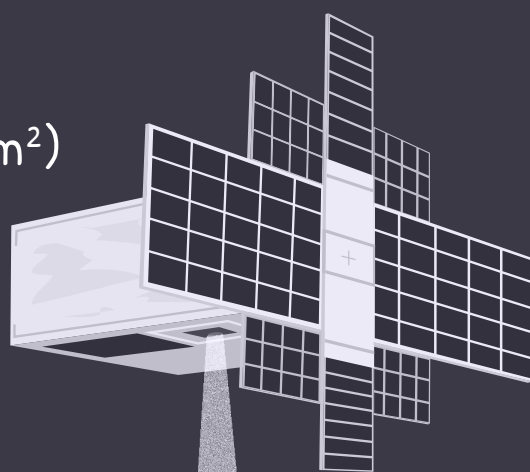
1. Use the formula for area of a circle to calculate the area covered by a laser pulse.

$$A = \pi r^2$$
$$A = \pi (17.5\text{m})^2$$
$$A = \pi (306.25 \text{ m}^2)$$
$$A \approx \boxed{962 \text{ m}^2}$$

Core Conundrum

What is the density of Mars' core?

1. Convert km to cm.

$$1{,}830 \text{ km} \bullet (100{,}000 \text{ cm} / 1 \text{ km}) = 183{,}000{,}000 \text{ cm} = 1.83 \bullet 10^8 \text{ cm}$$

2. Calculate the volume of Mars' core.

$$V = 4/3 \pi r^3$$
$$V = 4/3 \pi (1.83 \bullet 10^8 \text{ cm})^3$$
$$V \approx 4/3 \pi (6.13 \bullet 10^{24} \text{ cm})$$
$$V \approx 2.57 \bullet 10^{25} \text{ cm}^3$$

3. Convert kg to g.

$$(1.54 \bullet 10^{23} \text{ kg}) \bullet (1{,}000 \text{ g} / 1 \text{ kg}) = 1.54 \bullet 10^{26} \text{ g}$$

4. Divide the mass of Mars' core by its volume.

$$(1.54 \bullet 10^{26} \text{ g}) / (2.57 \bullet 10^{25} \text{ cm}^3) \approx \boxed{5.99 \text{ g/cm}^3}$$

How does that compare to the density of Earth's core?

Mars' core is less dense.

What does that tell us about the makeup of Mars' core?

Mars' core is made of less dense material than Earth's core.

Dam Deduction

Compute the velocity of the powered outflow.

1. Plug in the values for the measured height of the reservoir (H) above the penstocks and acceleration of gravity constant (g), and compute.

$$V = \sqrt{2gH} = \sqrt{2(9.8 \text{ m/s}^2)(100 \text{ m})} = \sqrt{1{,}960 \text{ m}^2/\text{s}^2} \approx \boxed{44 \text{ m/s}}$$

What is the powered outflow if 1 penstock is open?

1. Compute the area of the penstock opening and multiply by the velocity.

$$A = \pi r^2 = 3.14 \bullet (3.1 \text{ m})^2 \approx 30 \text{ m}^2$$
$$\text{Powered outflow} \approx (30 \text{ m}^2)(44 \text{ m/s}) \approx \boxed{1{,}320 \text{ m}^3/\text{s}}$$

Is this a high or low percentage of the total outflow?

1. Compute the ratio of powered outflow to total outflow.

$$(1{,}320 \text{ m}^3/\text{s}) / (1{,}350 \text{ m}^3/\text{s}) \approx 98\%$$

a high percentage of total outflow

What can this tell you about the potential environmental impacts?

The potential for environmental impact is high.

Telescope Tango

How many kilometers would TESS need to travel to successfully transmit its data if its velocity stayed uniform?

1. Plug in the values for the semi-major axis (apogee axis/2) and the semi-minor axis (perigee axis/2) into the equation for the perimeter of an ellipse to find the total distance TESS travels throughout its orbit.

$$P \approx \pi \left[3(a + b) - \sqrt{(3a + b)(a + 3b)} \right]$$
$$P \approx \pi \left[3(188{,}000 \text{ km} + 54{,}200 \text{ km}) - \right.$$
$$\left. \sqrt{((3 \bullet 188{,}000 \text{ km}) + 54{,}200 \text{ km}) \bullet (188{,}000 \text{ km} + (3 \bullet 54{,}200 \text{ km}))} \right]$$
$$P \approx 820{,}100 \text{ km}$$

2. Divide the downlink time by the time it takes TESS to complete its orbit to find the percentage of the orbit spent sending data back to Earth.

$$3 \text{ hours} / (13.7 \text{ days} \bullet 24 \text{ hours}) \approx 0.9\% \text{ of perimeter}$$

3. Multiply the percent of time transmitting by the total perimeter to get the distance covered in this time:

$$0.009 \bullet 820{,}100 \text{ km} \approx \boxed{7{,}380 \text{ km}}$$

Note: There are many ways to solve this problem. One way is to use the Ramanujan approximation as shown above. However, calculus can also be used.