



π IN THE SKY⁸

Answer Key

Sample Science

How many pads needed to make contact with Bennu's surface to meet the mission requirement?

1. Compute the area of each sample pad.

$$A = \pi r^2$$

$$\pi(0.75 \text{ cm})^2 \approx 1.8 \text{ cm}^2$$

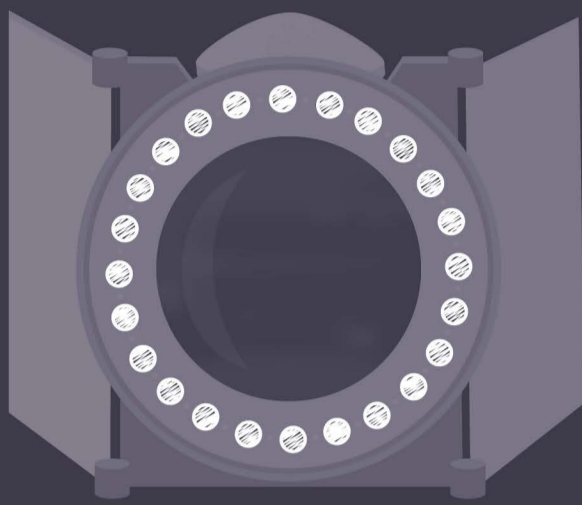
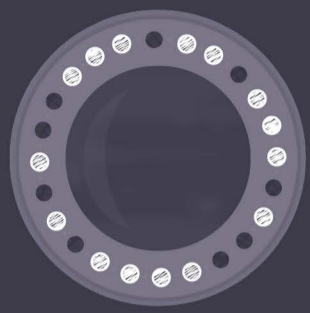
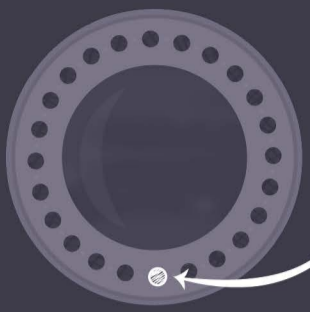
2. Divide the mission requirement for contact with Bennu's surface by the area of the sample pad.

$$26 \text{ cm}^2 \div (1.8 \text{ cm}^2/\text{pad}) \approx \text{15 pads}$$

If all 24 pads contacted Bennu, how much asteroid surface area would the contact pads sample?

1. Multiply the number of pads by the surface area contacted by one pad.

$$24 \text{ pads} \cdot (1.8 \text{ cm}^2/\text{pad}) \approx \text{43 cm}^2$$



Whirling Wonder

How fast, in rotations per minute, do Ingenuity's blades spin?

1. Convert radians to rotations per minute (1 rotation = 2π radians).

$$(250 \text{ rad / sec}) \cdot (60 \text{ sec / 1 min}) \cdot (1 \text{ rotation / } 2\pi \text{ radians})$$

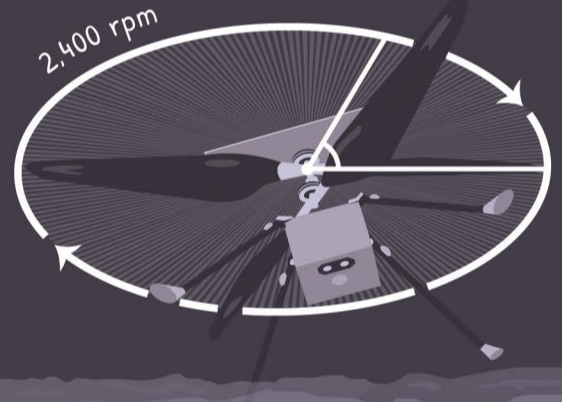
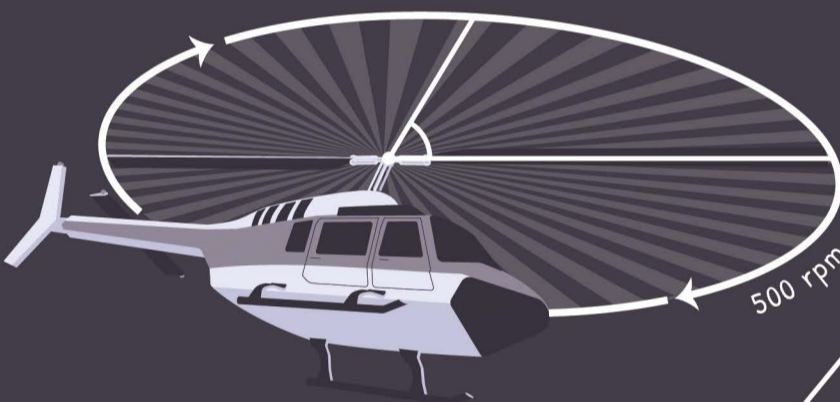
$$\approx \text{2,400 rpm}$$

How does that compare to a typical helicopter on Earth?

1. Divide Ingenuity rotations per minute by Earth helicopter rotations per minute.

$$2,400 \text{ rpm} / 500 \text{ rpm} = 4.8$$

Ingenuity's blades spin ~ 5 times faster



Signal Solution

What fraction of Voyager's original signal is received by a 70 meter antenna on Earth?

1. Convert astronomical units to meters.

$$124 \text{ AU} \cdot (150,000,000 \text{ km} / 1 \text{ AU}) \cdot (1,000 \text{ m} / 1 \text{ km}) = 1.86 \cdot 10^{13} \text{ m}$$

2. Find the beam radius at Earth using tangent and the distance between Earth and Voyager.

$$\tan 0.25^\circ \approx x / (1.86 \cdot 10^{13} \text{ m}) \rightarrow 8.12 \cdot 10^{10} \text{ m}$$

3. Find the ratio of the antenna area (radius of 35 m) to the signal area.

$$\pi(35 \text{ m})^2 / \pi(8.12 \cdot 10^{10} \text{ m})^2 \approx 1.9 \cdot 10^{-19}$$

4. Find the ratio of received signal versus the sent signal.

$$1.9 \cdot 10^{-19} \cdot 12.5 \text{ W} \approx \text{2.3} \cdot 10^{-18} \text{ W or } 1.8 \cdot 10^{-20} \%$$

What fraction of the signal from a DSOC-equipped spacecraft is received?

1. Follow the same process as above with the values for the DSOC-equipped spacecraft.

$$24 \text{ AU} \cdot (150,000,000 \text{ km} / 1 \text{ AU}) \cdot (1,000 \text{ m} / 1 \text{ km}) \approx 1.86 \cdot 10^{13} \text{ m}$$

$$\tan 0.00045^\circ \approx x / (1.86 \cdot 10^{13} \text{ m}) \rightarrow 1.46 \cdot 10^8 \text{ m}$$

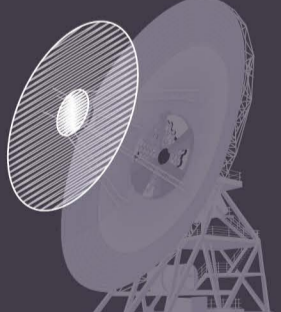
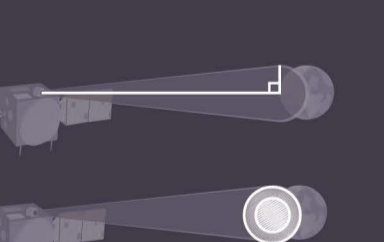
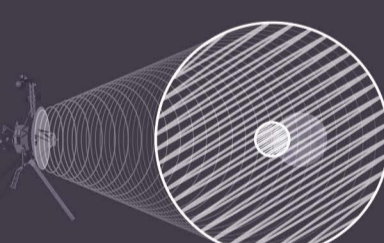
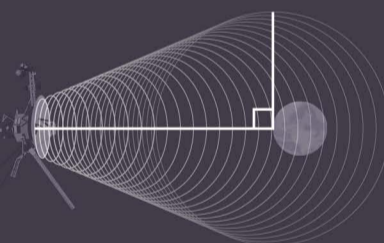
$$\pi(35 \text{ m})^2 / \pi(1.46 \cdot 10^8 \text{ m})^2 \approx 5.7 \cdot 10^{-14}$$

$$5.7 \cdot 10^{-14} \cdot 4 \text{ W} \approx \text{2.3} \cdot 10^{-13} \text{ W or } 5.8 \cdot 10^{-14} \%$$

By what factor is DSOC more effective?

1. Divide the received wattage of the DSOC spacecraft's signal by that of Voyager's.

$$2.3 \cdot 10^{-13} / 2.3 \cdot 10^{-18} = \text{10}^5 \text{ or } 100,000 \text{ times more effective}$$



Force Field

What force does a hydrogen ion at π/4 radians from the equator observe? What about at the North Pole (π/2 radians)?

1. Convert microteslas to teslas and kilometers per second to meters per second.

$$60 \mu\text{T} = 6 \cdot 10^{-5} \text{ T}$$

$$400 \text{ km/s} = 4 \cdot 10^5 \text{ m/s}$$

2. Enter the known values into the Lorentz force equation and compute.

$$F = (1.602 \cdot 10^{-19} \text{ C}) \cdot (4 \cdot 10^5 \text{ m/s}) \cdot (6 \cdot 10^{-5} \text{ T}) \cdot \sin(\pi/4)$$

$$F \approx \text{3} \cdot 10^{-18} \text{ N}$$

$$F = (1.602 \cdot 10^{-19} \text{ C}) \cdot (4 \cdot 10^5 \text{ m/s}) \cdot (6 \cdot 10^{-5} \text{ T}) \cdot \sin(\pi/2)$$

$$F \approx \text{4} \cdot 10^{-18} \text{ N}$$

Does the relative magnetic field agree or disagree with what you'd expect about the location of auroras?

1. **Agrees.** A larger Lorentz force occurs at the North Pole where the formation of auroras is more common.

