π IN THE SKY⁷

Answer Key

Coral Calculus

Using the absorption coefficient and Beer-Lambert law formulas, calculate the water's depth.

Estimate distance on the blue and red ends of the spectrum:

1. Solve for the blue light and red light absorption coefficients.

$$\alpha = (4\pi k) / \lambda$$

blue light: $\alpha = (4\pi \cdot 1.01\text{E}{-}09) / (0.00000045 \text{ m}) \approx 0.028/\text{m}$

red light: $\alpha = (4\pi \cdot 1.60\text{E}{-}08) / (0.00000065 \text{ m}) \approx 0.309/\text{m}$

2. Rearrange the Beer-Lambert law formula, $T = e^{(-\alpha \cdot d)}$, to solve for d.

$$\ln(T) = \ln(e^{(-\alpha \cdot d)})$$
$$\ln(T) = -\alpha \cdot d$$
$$d = \ln(T) / (-\alpha)$$

3. Solve for d on the blue and red ends of the spectrum.

blue light: $d = \ln(0.76) / (-0.028) \approx 9.73 \text{ m}$

red light: $d = \ln(0.045) / (-0.309) \approx 10.04 \text{ m}$

4. Because light passes through the water twice, divide the total distances by 2.

blue light: $9.73 \text{ m}/2 \approx 4.87 \text{ m}$

red light: $10.04 \text{ m}/2 \approx 5.02 \text{ m}$

5. Find the weighted mean of the distances from both ends of the spectrum.

$$((0.76 \cdot 4.87) + (0.045 \cdot 5.02)) / (0.76 + 0.045) \approx \boxed{5 \text{ m}}$$

5 m