**Answer Key**

**Pi in the Sky 7: Coral Calculus**

Flying aboard an aircraft, NASA’s CORAL mission uses spectroscopy to study the health of coral reefs and the threats they face. To differentiate among coral, algae and sand on the ocean floor, CORAL computes the depth of every point it maps. The water’s depth is determined using the “absorption coefficient,” how much light is absorbed through a given depth of water.

Imagine CORAL collects a light measurement reflected by white sand covered by an unknown depth of water that is 76% in the blue and 4.5% in the red. **Using the formulas below, calculate the water’s depth.** Note that the sunlight passes through the water twice: when traveling from the Sun to the ocean floor and when reflecting up to the aircraft.

**absorption coefficient, α** = **(4•Pi•k)/λ**

**k** = coefficient of the imaginary number portion of the refractive index

**λ** = wavelength (in meters) of light observed

**Beer-Lambert law, T** = **exp(-α•d)**

**T** = observed reflectance, or transmittance (T) of light through a distance (d) of water

**For water in the blue wavelength (450 nm) the refractive index = 1.3369 + 1.01E-09i**

**For water in the red wavelength (650 nm) the refractive index = 1.3314 + 1.60E-08i**

**Solution**

1. Solve for the blue light and red light absorption coefficients.

blue light: α = (4πk) / λ = (4π • 1.01E-09) / (0.00000045 m) ≈ 0.028/m

red light: α = (4π • 1.60E-08) / (0.00000065 m) ≈ 0.309/m

1. Rearrange the Beer-Lambert law formula, T = e^(-α•d), to solve for d.

ln(T) = ln(e^(-α•d))

lnT = -α•d

d= lnT / (-α)

(Cont. on next page)

1. Solve for d on the blue and red ends of the spectrum.

blue light: d = ln (0.76) / (-0.028) ≈ 9.73 meters

red light: d = ln(0.045) / (-0.309) ≈ 10.036 m

1. Because light passes through the water twice, divide the total distances by 2.

blue light: 9.73 m / 2 ≈ 4.87 meters

red light: 10.04 m / 2 ≈ 5.02 meters

1. Find the weighted mean of the distances from both ends of the spectrum.

((0.76 \* 4.87) + (0.045 \* 5.02)) / (0.76 + 0.045) ≈ **5 m**