**Answer Key**

**Pi in the Sky 7**

**MARS MANEUVER**

When we plan where to land a spacecraft on Mars, we don’t choose a specific spot, but a larger area called a landing ellipse. It's like choosing a parking lot rather than a parking spot. To choose a landing ellipse, we have to compromise between getting as close as possible to interesting science targets and avoiding hazards. As we've created new technology to help direct spacecraft, landing ellipses have gotten smaller and smaller. That means that we're able to land in places we couldn't before and get closer to the stuff we want to study.

In 2012, the Curiosity rover used its sky crane landing system to touch down in a 20 km by 7 km ellipse. When the Mars 2020 rover lands on Feb. 18, 2021, it will use the same system along with a new technique called Range Trigger that will allow the spacecraft to land in the smallest ellipse yet, measuring just 13 km by 7 km. **What percentage of Curiosity's landing ellipse is Mars 2020's landing ellipse?**

**Solution**

1. Divide the area of Perseverance’s landing ellipse by the area of Curiosity’s landing ellipse, using the formula for the area of an ellipse. (Note: π cancels π.)

Aellipse = πab

(π \* 3.5 \* 6.5) / (π  \* 3.5 \* 10) → (3.5 \* 6.5) / (3.5 \* 10) = 0.65

1. Convert 0.65 to a percentage

0.65 \* 100 = **65%**

**COLD CASE**

In January 2019, NASA's New Horizons spacecraft flew within 3,538 km of the most distant and primitive object explored up-close by a spacecraft. The object was originally known as 2014 MU69, but it was later renamed Arrokoth. It looks like a partially flattened, reddish snowman and is made up of two objects that merged into one. Found 6.6 billion km from Earth, Arrokoth is a small “Cold Classical” Kuiper Belt object, meaning it orbits the Sun in a nearly-circular path and has a low orbital inclination. Cold Classical objects make up about one-third of the Kuiper Belt.

One reason scientists are interested in studying Arrokoth and other Kuiper Belt objects is that they are thought to be well preserved, frozen samples of what the outer solar system was like at its birth, more than 4.5 billion years ago. **Learn a bit more about Arrokoth by calculating how long it takes the object to make one trip around the Sun.**

**Solution**

1. Using the pi formula for circumference, compute distance traveled by Arrokoth in one orbit.

C = 2π r

C = 2π (6,600,000,000 km + 150,000,000 km)

C = 2π (6,750,000,000 km)

C ≈ 42,411,500,823 km

1. Convert radius kilometers to meters, then compute Arrokoth’s orbital velocity.

V = (GMSun)/r)

V=√((6.67x10^(-11) m^3kg^-1s^-2)\*(2x10^30 kg)) / 6.75 x 10^12 m)

V ≈ 4,446 m/s

1. Convert circumference kilometers to meters, then use d=rt to compute the time it takes Arrokoth to complete one orbit.

t ≈ (42,411,500,823,000 m)/(4,446 m/s) ≈ 9,539,248,948 s

1. Convert seconds to years.

(9,539,248,948 s) \* (1 min/60 s) \* (1 hour/60 min) \* (1 day/24 hours) \* (1 year/365 day)

≈ **300 years**

**CORAL CALCULUS**

Flying aboard an aircraft, NASA’s CORAL mission uses spectroscopy to study the health of coral reefs and the threats they face. To differentiate among coral, algae and sand on the ocean floor, CORAL computes the depth of every point it maps. The water’s depth is determined using the “absorption coefficient,” how much light is absorbed through a given depth of water.

Imagine CORAL collects a light measurement reflected by white sand covered by an unknown depth of water that is 76% in the blue and 4.5% in the red. **Using the formulas below, calculate the water’s depth.** Note that the sunlight passes through the water twice: when traveling from the Sun to the ocean floor and when reflecting up to the aircraft.

**absorption coefficient, α** = **(4•Pi•k)/λ**

**k** = coefficient of the imaginary number portion of the refractive index

**λ** = wavelength (in meters) of light observed

**Beer-Lambert law, T** = **exp(-α•d)**

**T** = observed reflectance, or transmittance (T) of light through a distance (d) of water

**For water in the blue wavelength (450 nm) the refractive index = 1.3369 + 1.01E-09i**

**For water in the red wavelength (650 nm) the refractive index = 1.3314 + 1.60E-08i**

**Solution**

1. Solve for the blue light and red light absorption coefficients.

blue light: α = (4πk) / λ = (4π • 1.01E-09) / (0.00000045 m) ≈ 0.028/m

red light: α = (4π • 1.60E-08) / (0.00000065 m) ≈ 0.309/m

1. Rearrange the Beer-Lambert law formula, T = e^(-α•d), to solve for d.

ln(T) = ln(e^(-α•d))

lnT = -α•d

d= lnT / (-α)

1. Solve for d on the blue and red ends of the spectrum.

blue light: d = ln (0.76) / (-0.028) ≈ 9.73 meters

red light: d = ln(0.045) / (-0.309) ≈ 10.036 m

1. Because light passes through the water twice, divide the total distances by 2.

blue light: 9.73 m / 2 ≈ 4.87 meters

red light: 10.04 m / 2 ≈ 5.02 meters

1. Find the weighted mean of the distances from both ends of the spectrum.

((0.76 \* 4.87) + (0.045 \* 5.02)) / (0.76 + 0.045) ≈ **5 m**

**PLANET PINPOINTER**

Our galaxy contains billions of stars, most of which are likely home to exoplanets – planets outside our solar system. How do scientists decide where to look for these worlds? Researchers looking at data from NASA's Spitzer Space Telescope found that giant exoplanets tend to exist around young stars surrounded by a disk of debris.

A prominent debris disk around the star Beta Pictoris, which is 5.99864 x 1014 km away from Earth, lead scientists to discover two exoplanets several times bigger than Jupiter orbiting the star! Learning more about Beta Pictoris’ debris disk could give scientists insight into the formation of these giant worlds. Given the angle of the disk's apparent size is 169 arcseconds, determine the actual distance across it using the formula for small angle approximation, below. (An arcsecond is 1/3,600 of a degree.)

**D=dθ**

**D** = distance across the debris disk (in km)

**d** = distance to Beta Pictoris (in km)

**Θ** = angle of apparent size (in radians)

**Solution**

1. Convert arcseconds to degrees.

1 arcsec = (1/3,600)°

169 arcsec \* 1° / 3,600 arcsec ≈ 0.0469°

1. Multiply degrees π/180° to convert degrees to radians

0.0469° \* (π / 180°) ≈ 0.000819 radians

1. Use the formula for small angle approximation to find the distance across the Beta Pictoris debris disk.

D = dθ

D = (6\*10^14) \* 0.000819 ≈ **500 billion km**