

# π IN THE SKY<sup>5</sup>

By now, you know a slice of pi can take you far. Check your answers below to see if your pi skills could one day make you a NASA space explorer.

## Solar Sleuth

What is the radius of the exoplanet, known as Kepler-186f?

$A_p$  = disk area of the planet  
 $A_s$  = disk area of the star

1. Multiply the dip in brightness by the star's disk area to find the disk area of Kepler-186f.

$$A_p = 0.00042 \cdot 416,000,000,000 \text{ km}^2 = 174,720,000 \text{ km}^2$$

2. Use the disk area of Kepler-186f and the pi formula for the area of a circle to find the planet's radius.

$$A_p = \pi r_p^2$$

$$174,720,000 \text{ km}^2 = \pi r_p^2$$

$$\left( \frac{174,720,000 \text{ km}^2}{\pi} \right) = r_p^2$$

$$r_p^2 \approx 55,615,103 \text{ km}^2$$

$$r_p \approx \sqrt{55,615,103 \text{ km}^2}$$

$$r_p \approx 7460 \text{ km}$$

## Helium Heist

If 10% of the helium volume in Jupiter's molecular hydrogen layer has been rained out since the planet formed, what is the volume in cubic km that has rained out?

1. Use the formula for the volume of a sphere to find the volume of Jupiter.

$$4/3 \cdot \pi \cdot (70,000 \text{ km})^3 \approx 1,440,000,000,000,000 \text{ km}^3$$

2. Find the volume of the inner 50,000 km of Jupiter.

$$4/3 \cdot \pi \cdot (50,000 \text{ km})^3 \approx 524,000,000,000,000 \text{ km}^3$$

3. Subtract the volume of the innermost 50,000 km from the total volume of Jupiter to find the volume of the molecular hydrogen layer.

$$1,440,000,000,000,000 \text{ km}^3 - 524,000,000,000,000 \text{ km}^3 = 916,000,000,000,000 \text{ km}^3$$

4. Find 1% (10% of 10%) of the volume of the molecular hydrogen layer.

$$916,000,000,000,000 \text{ km}^3 \cdot 0.01 = 9,160,000,000,000 \text{ km}^3$$

Given that Earth's radius is 6,371 km, about how many Earth-size spheres of helium is that?

1. Use the formula for the volume of a sphere to find the volume of Earth.

$$4/3 \cdot \pi \cdot (6,371 \text{ km})^3 \approx 1,083,000,000,000 \text{ km}^3$$

2. Divide the volume of 1% of Jupiter's molecular hydrogen layer by Earth's volume.

$$9,160,000,000,000 \text{ km}^3 / 1,083,000,000,000 \text{ km}^3 \approx 8 \text{ Earth volumes}$$

## Quake Quandary

Given the times that surface waves were recorded by NASA's Mars InSight lander, find the velocity (U) in rad/s of the surface wave, the distance in radians on the sphere from InSight to the epicenter ( $\Delta$ ), and the time the marsquake occurred ( $t_0$ ).

1. Convert all surface wave times to seconds.

$$R_1 = 08:38:09.4 \Rightarrow$$

$$(8 \text{ hr} \cdot 3,600 \text{ sec/hr}) + (38 \text{ min} \cdot 60 \text{ sec/min}) + 9.4 \text{ sec} = 31,089.4 \text{ sec}$$

$$R_2 = 10:04:48.2 \Rightarrow$$

$$(10 \text{ hr} \cdot 3,600 \text{ sec/hr}) + (4 \text{ min} \cdot 60 \text{ sec/min}) + 48.2 \text{ sec} = 36,288.2 \text{ sec}$$

$$R_3 = 10:25:43.0 \Rightarrow$$

$$(10 \text{ hr} \cdot 3600 \text{ sec/hr}) + (25 \text{ min} \cdot 60 \text{ sec/min}) + 43 \text{ sec} = 37,543 \text{ sec}$$

2. Plug the times into the provided formulas.

$$U = \frac{2\pi}{(R_3 - R_1)} = \frac{2\pi}{(37,543 - 31,089.4)} = \frac{2\pi}{6,453.6} \approx 0.00097 \text{ rad/sec}$$

$$\Delta = \pi - \frac{U(R_2 - R_1)}{2} = \pi - \frac{U(36,288.2 - 31,089.4)}{2} \approx \pi - 2.5308 \approx$$

$$0.61083 \text{ rad}$$

$$t_0 = R_1 - \frac{\Delta}{U} = 31,089.4 - \frac{\Delta}{U} \approx 30,462 \text{ sec} \Rightarrow 08:27:42 \text{ UTC}$$

## Asteroid Ace

Based on the findings, what is the angular rotation rate of asteroid 'Oumuamua in rad/s?

1. Use dimensional analysis to convert 'Oumuamua's rotation rate to radians per second.

$$\frac{1 \text{ rotation}}{7.3 \text{ hours}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rotation}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}}$$

$$\frac{1 \text{ rotation}}{7.3 \text{ hours}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rotation}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}}$$

$$= \frac{2\pi \text{ radians}}{26,280 \text{ seconds}} \approx 0.00024 \text{ rad/sec}$$

How does this compare with Earth's rotation rate?

1. Divide Earth's rotation rate in hours by 'Oumuamua's.

$$24/7.3 \approx \text{'Oumuamua rotates 3.29 times faster than Earth!}$$